CS 381 Final Exam

December, 2010

1. State the following formulas in English, where the universe is the set of objects and the meaning of the predicate symbols are as follows:[15]

$B(x)$ means x is a book, $I(x)$ means x is interesting, $P(x)$ means x is a person, and $BY(x, y)$ means x buys y.

(a) $\exists x[B(x) \wedge I(x)]$

(b) $\exists x[B(x) \rightarrow I(x)]$

(c) $\exists x \forall y[B(x) \wedge [P(y) \rightarrow BY(y, x)]]$

(d) $\forall x[[B(x) \wedge I(x)] \rightarrow \forall y[P(y) \rightarrow BY(y, x)]]$

(e) $\forall x \exists y[B(x) \rightarrow [P(y) \wedge BY(y, x)]]$

2. Recursively define each of the following (a) and (b):

(a) $\cap_{i=1}^{n} A_i$, where $n \geq 2$ and n is a natural number. [7 Pts.]

(b) The binary relation $R = \{< a, b >| \ a < b + 1$, where a and b are natural numbers $\}$ [8 Pts.]

3. For the following statement answer the questions (a) and (b) below:

For sets A_i, $1 \leq i \leq n$, where i and n are natural numbers and $n \geq 2$, if $A_i \subseteq A_{i+1}$ for all $i, 1 \leq i \leq n-1$, then $A_1 = \cap_{i=1}^{n} A_i$.

(a) State the given statement for $n = 2$ [5 Pts.]

(b) Prove the given statement by mathematical induction. [10 Pts.]

4. Prove the following statements by mathematical induction, where n is a natural number:

(a) $\Sigma_{i=1}^{n} 1/i(i+1) = n/(n+1)$ [7 Pts.]

(b) $2^n > n^3$ for $n \geq 10$. [8 Pts.]

5 (a) For the subset relation over the power set of $\{1, 2\}$, draw a Hasse diagram. [5 Pts.]

(b) For the relation of (a) find a topological order. [5 Pts.]

(c) How many topological orders are possible for the relation of (a) ? [5 Pts.]

(d) Prove that the subset relation over the power set of a set is a partial order. [10 Pts.]

6. Which of the following statements are true and which are false ? [15 Pts.]

(a) $x^2 + 2x = O(3x^2 - x + 2)$

(b) $10x^2 + 100x = O(x^3 - 100x - 1000)$

(c) $2^x = O(x^{10})$

(d) For any partition there is an equivalence relation corresponding to it.

(e) If a relation is transitive and symmetric, then it is reflexive.

(f) Every function has an inverse function.

(g) Functions f and g must be bijections if the composite function gf is one-to-one.

(h) For sets A and B, if $A - B = B - A$ then $A = B$.

(i) $((P \to Q) \wedge Q) \Rightarrow P$

(j) $\forall x(P(x) \wedge Q(x)) \Leftrightarrow (\forall x P(x) \wedge \forall y Q(y))$