CS 381 Final Exam

December 2009

1. Prove for sets A, B and C that $(A - C) - (B - C) \subseteq (A - B)$ [10]

2(a) Recursively define the set P of even integers. [10]

(b) Recursively define the set $Q = \{< x, y >:$ x and y are nonnegative integers and $x + y$ is even$\}$. [10]

3(a) Prove by mathematical induction that $\Sigma_{i=0}^{n}(2i+1) = (n+1)^2$ [10]

(b) Prove for propositions $P_1, P_2, ..., P_n$ by mathematical induction that $((P_1 \rightarrow P_2) \wedge (P_2 \rightarrow P_3) \wedge ... \wedge (P_{n-1} \rightarrow P_n)) \rightarrow ((P_1 \wedge P_2 \wedge P_3 \wedge ... \wedge P_{n-1}) \rightarrow P_n)$ [10]

You may use $(X \rightarrow Y) \Rightarrow ((X \wedge Z) \rightarrow (Y \wedge Z))$, and $((X \rightarrow Z) \wedge (Y \rightarrow Z)) \Rightarrow (X \wedge Y) \rightarrow Z$ for any propositions X, Y, and Z.

4. Prove that if R is an antisymmetric and transitive relation, then R^2 is antisymmetric. [10]

5. Let $S = \{x : 0 \leq x \leq 11, \text{ and } x \text{ is an integer}\}$.
Let R be the relation on S such that xRy iff $x \equiv y \bmod 4$, that is iff $\mid x - y \mid$ is a multiple of 4.

(a) Prove that R is transitive. [10]

(b) List up all the equivalence classes of R. [5]

(c) Draw the graph of R for the equivalence class of 3. [5]

6. For each of the following statements state whether or not it is true.[20]

(a) It is necessary for $x^4 \geq 0$ that $x \geq 0$.

(b) The negation of "Everyone has read every section of every chapter of the book" is "Someone has not read some section of every chapter of the book".

(c) The contrapositive of "It is necessary for me to walk 8 miles in order to get to the top of the mountain" is "If I don't go to the top of the mountain, then I don't walk 8 miles".

(d) $\emptyset \in \{\emptyset, \{1\}\}$.

(e) $\emptyset \subseteq \{\emptyset, \{1\}\}$.

(f) $y = x^2$ is a bijection on the set of natural numbers.

(g) If $f(x)$ and $g(x)$ are one-to-one function from a set A of natural numbers to a set B of natural numbers, then $f(g(x))$ is a bijection from A to B.

(h) If $A - B = B - A$ for sets A and B, then $A = B$.

(i) $2^{\emptyset} = \emptyset$

(j) If $\forall x \exists y \exists z[(x, y) \in R \wedge (z, x) \in R]$, then the transitive closure of R is an equivalence relation.