CS 381 Final Exam

December 2007

1 (a) Express the argument given below as propositions of **predicate logic** using the following symbols: [6]

$C(x)$: x is in this class.
$S(x)$: x likes skiing.
$W(x)$: x likes winter.

The universe is the set of people in the world.

(b) Check whether or not the reasoning is correct. Give your reasons. [9]

Argument:
John who is a student in this class likes winter. Everyone who likes winter likes skiing. Therefore someone in this class likes skiing.

2 (a) Recursively define the set P of odd positive integers. [7]

Let $S = \{ < a, b >| \; a, b$ are positive integers and $a + b$ is odd $\}$ and answer the following questions:

(b) Give two examples of elements of S. [4]

(c) Define S recursively. [8]

3. Prove by mathematical induction the following:

(a) $\Sigma_{i=1}^{n} 1/(i(i+1)) = n/(n+1)$ [10]

(b) $(A_1 - B) \cup (A_2 - B) \cup ... \cup (A_n - B) = (A_1 \cup A_2 \cup ... \cup A_n) - B$
i.e. $\cup_{i=1}^{n}(A_i - B) = (\cup_{i=1}^{n} A_i) - B$. [10]

4. Let S be the set of strings consisting of one or more a's and/or b's such as a, b, aa, baa, bbab, etc.
S can be defined recursively as follows:

Basis Clause: $a \in S, b \in S$.
Inductive Clause: If $x \in S$, then $xa \in S$ and $xb \in S$, where xa is the string obtained by appending a to x.
Extremal Clause: Nothing is in S unless it is obtained by the above Basis and Inductive Clauses.

Let $\leq$ be the lexicographic order on the set S defined above. Answer the following questions:

(a) Prove that $(S, \leq)$ is a poset i.e. $\leq$ is a partial order on S. [10]

(b) Prove that $\leq$ on S is in fact a total (linear) order. [8]

Let T be the subset of S defined above consisting of all the strings of three or less but at least one characters and answer the following questions:

(c) List all the strings of T. [5]

(d) Draw the Hasse diagram of $(T, \leq)$. [5]

5. Let $f(x) = 2x + 1$ be a function from the set N of natural numbers to the set P of positive odd integers.

(a) Find the inverse function f^{-1} of f (No proof is necessary). [8]

(b) Prove that f is a bijection (i.e. a one-to-one, onto function). [10]