CS 381 Final Exam

July, 2006

1. For each of the following sentences answer the questions given below:

(1) Everyone can fool Fred.
(2) Evelyn can fool everyone.
(3) Everyone can fool someone.
(4) There is someone who can fool everyone.
(5) If someone can fool everyone, then everyone can fool everyone.

(a) Express it as a proposition of a predicate logic using $F(x, y)$ to mean x can fool y. The universe is the set of people.[10]

(1)

(2)

(3)

(4)

(5)

(b) Negate it in English. DO NOT simply say "It is not the case that ..." or something similarly trivial. [10]

(1)

(2)

(3)

(4)

(5)

2. Recursively define each of the following sets:

(a) The set of strings of 1's of odd length (denote it L_1) i.e. $\{1, 111, 11111, ...\}$. You may use concatenation operation in your definition. [10]

(b) The set of natural numbers that are NOT divisible by 3 (denote it L_2). [10]

3. Prove each of the following by mathematical induction:

(a) $\Sigma_{i=1}^{n} 1/i(i+1) = n/(n+1)$. [10]

(b) By concatenating 11's and/or 111's, all lengths of strings of 1's except 1 can be obtained. Note that the length of string of 1's is the number of 1's in the string e.g. length of 11 is 2, length of 111 is 3 etc. [10]

4. Prove that $(A - B) - C \subseteq (A - C)$ by showing that for an arbitrary element x if $x \in (A - B) - C$ then $x \in (A - C)$. [10]

5. Let us define a binary relation R on the set of strings of length 3 or longer consiting of 1's and 0's as

$x \ R \ y$ if and only if x and y agree in the first three symbols (i.e. the first three positions from left).
For example 1001 R 1100 and 11010 R 11011 are true but 11010 R 10010 is not true.
Now answer the following questions:

(a) Prove that R is an equivalence relation. [10]

(b) What are equivalence classes of R ? Also give an example. [5]

6. Which of the following statements are true and which are false ? [?] f is a function from set A to set B and g is a function from B to A. [15]

(a) If gf is a function, then fg is onto.

(b) If f is not one-to-one, then gf is not one-to-one.

(c) If f is one-to-one, then gf is one-to-one.

(d) If f is onto, then gf is onto.

(e) If $g(f(x)) = x$, then f is onto.