CS 381 Test

July, 2006

1. Fill in the blanks with the **SHORTEST** string of characters so that the resultant proposition is valid. [20]

(a) $\neg(P \rightarrow Q) \Leftrightarrow \neg(\neg P$ [$\vee$] $Q)$
$\Leftrightarrow P$ [$\wedge$] [$\neg Q$]

(b) $(P \vee (P \wedge Q) \Leftrightarrow (P \wedge$ [T] $) \vee (P \wedge Q)$
$\Leftrightarrow P \wedge ($ [T] [$\vee$] $Q)$
$\Leftrightarrow P \wedge$ [T] $\Leftrightarrow P$

(c) $P \rightarrow (Q \wedge R) \Leftrightarrow$ [$\neg P$] $\vee (Q \wedge R)$
$\Leftrightarrow \neg P \vee \neg(\neg(Q$ [$\wedge$] $R))$
$\Leftrightarrow \neg(Q$ [$\wedge$] $R) \rightarrow \neg P$

2. State each of the following formulas **in English**, **if** it is a wff. If it is not a wff, then give a reason why it is not a wff. Here $L(x, y)$ means x is larger than y, $I(x)$ means x is an integer and the universe is the set of real numbers: [12]

(a) $\forall x \neg \exists y L(x, y)$

For every number, there is no number that is less than that number.
i.e. Every number is not greater than nany number.

(b) $\forall x \exists y \neg L(x, y)$

For every number there is a number that is not less than that.
i.e. Every number is not greater than some number.

(c) $\forall x \exists y[[I(x) \wedge I(y)] \rightarrow L(x, y)]]$

For every number x there is a number y such that if x and y are integer, then x is gretaer than y.
i.e. For every integer, there is an integer that is less than that.
i.e. Every integer is greater than some integer.

(d) $\forall x \exists I(y) L(I(x), y)$

This is not a proposition (well-formed formula) because an atomic formula $I(x)$ is an argument of another atomic formula $L(x, y)$.

3. Negate the following sentences in English. DO NOT simply say "It is not the case that ..." or something similarly trivial. [12]

(a) Every number is greater than 0.

Some number is not greater than 0.

(b) Some number is greater than or equal to every number.

Every number is less than some number.

(c) Some numbers are even only if they are integer.

Every number is even and not integer.

4. Express the assertions given below as a proposition of a predicate logic using the following predicates. The universe is the set of numbers.[12]

$E(x)$: x is even.
$I(x)$: x is integer.

(a) Not all numbers are integer.

$\neg\forall x I(x)$

(b) A (every) number is even only if it is integer.

$\forall x[E(x) \rightarrow I(x)]$

(c) It is not necessary for a number to be integer that it is even.

$\neg\forall x[I(x) \rightarrow E(x)]$, which I prefer, or
$\forall x\neg[I(x) \rightarrow E(x)]$

5. Find the power set of each of the following sets: [9]

(a) $\{1, 5\}$

$\{\emptyset, \{1\}, \{5\}, \{1, 5\}\}$

(b) $\emptyset$

$\{\emptyset\}$

(c) { { 1 } , { $\emptyset$ } }

$\{\emptyset, \{\{1\}\}, \{\{\emptyset\}\}, \{\{1\}, \{\emptyset\}\}\}$

6. Indicate which of the following are true and which are false. [15]

(a) $\{1, 2\} \in \{1, 2, \{1, 2\}\}$ True
(b) $\{1\} \subseteq \{\{1\}\}$ False
(c) $\{1\} \in \{1, 2\}$ False
(d) $\emptyset \subseteq \{\emptyset\}$ True
(e) $\{\emptyset\} \in \{\emptyset\}$ False

7 (a) Express the argument given below as propositions of propositional logic using the symbols suggested for each proposition. [7]
(b) Check whether or not the reasoning is correct using inference rules on the wffs (symbolic form) of (a). Show your reasoning (in symbolic form). [13]

Argument: Either (if the browser is slow(B) then there may be some spywares in the system(S)), or (if the start up process is slow (U) then there may be some infected files in the system(I)). The start up process is not slow but the browser is slow. Therefore there are some spywares in the system.

(a) $(B \rightarrow S) \vee (U \rightarrow I)$
$\neg U \wedge B$

S

(b) Though you can deduce S from $(B \rightarrow S)$ and B, since $(B \rightarrow S)$ does not necessarily hold, S can not be deduced. Thus the reasoning is not correct.