CS 381 Test 2

April, 2005

1. Express the assertions given below as propositions of predicate logic using the following predicates. The universe is **the set of objects.**

$P(x)$: x is a person.
$S(x)$: x studies hard.
$T(x)$: x is a student.
$W(x)$: x will get sick.

(1) Everyone studies hard only if one is a student.

(2) If everyone studies hard, then someone will get sick.

(3) Everyone will get sick, if one studies hard.

2. State the following formula **in English**, where the universe is **the set of objects** and the meaning of the predicate symbols are as follows:

$A(x, y)$: x is attractive for y.
$B(x)$: x is a bee.
$E(x, y)$: x is easy to find for y.
$F(x)$: x is a flower.

(1) $\forall x[F(x) \rightarrow \exists y[B(y) \wedge A(x, y)]]$

(2) $\forall x \forall y[[F(x) \wedge B(y)] \rightarrow [A(x, y) \wedge E(x, y)]]$

(3) $\forall x[F(x) \rightarrow [\exists y[B(y) \wedge E(x, y)] \vee \exists y[B(y) \wedge A(x, y)]]]$

3(a). Give a rcursive definition of the set of palindromes of oddd length

consisting of symbols "a" and "b". A palindrome is a string that reads the same backwards as forwards. For example, "a", "aba". "bbbbb" and "baaab" are palindromes of odd length.

(b). Using your definition in 3(a), find all palindromes of length 1 and 3.

4. Which of the following statements are true and which are false ?

(1) $\{1,2\} \times \emptyset = \{<1,\emptyset>,<2,\emptyset>\}$
(2) $\{\emptyset\} \subseteq \{2\}$
(3) $\emptyset \in \{1,2\}$
(4) $\{1,2\} = \{1,2,1\}$ (5) $(A-B) \cup (A-C) = A - (B \cap C)$
(6) If $A \cup B = B$, then $A \cap B = A$
(7) $\{1\} \subseteq 2^{\{1,2\}}$
(8) $\{\emptyset\} \in A$ for every set A.
(9) If $A \cup B = U$ and $A \cap B = \emptyset$, where U is the universal set, then $B = \overline{A}$
(10) $(A-B) \cup (B-C) = A - C$
(11) If $A - B \cup (B-A) = A \cup B$, then $A \cap B = \emptyset$
(12) The cardinality (size) of $\{1,2,1\}$ is 3.
(13) $\mid A \times B \mid = 6$ if $\mid A \mid = 3$ and $\mid B \mid = 2$.
(14) The maximum possible value of $\mid A \cap B \mid$ is equal to the smaller of $\mid A \mid$ and $\mid B \mid$.
(15) $A \times B = B \times A$

5. Prove by mathematical induction the following:

$\Sigma_{i=1}^{n}(4i-3) = n(2n-1)$