CS 381 Solutions to Test 2

April, 2005

1. Express the assertions given below as propositions of predicate logic using the predicates given below. The universe is **the set of objects.** [15]

> $P(x)$: x is a person.
> $S(x)$: x studies hard.
> $T(x)$: x is a student.
> $W(x)$: x will get sick.

(1) Everyone studies hard only if one is a student.

$$\forall x[[P(x) \land S(x)] \rightarrow T(x)]$$

(2) If everyone studies hard, then someone will get sick.

$$\forall x[P(x) \rightarrow S(x)] \rightarrow \exists x[P(x) \land W(x)]$$

(3) Everyone will get sick, if one studies hard.

$$\forall x[[P(x) \land S(x)] \rightarrow W(x)]$$

2. State the formulas given below **in English**, where the universe is **the set of objects** and the meaning of the predicate symbols are as follows: [15]

> $A(x, y)$: x is attractive for y.
> $B(x)$: x is a bee.
> $E(x, y)$: x is easy to find for y.
> $F(x)$: x is a flower.

(1) $\forall x[F(x) \rightarrow \exists y[B(y) \land A(x, y)]]$

Every flower is attractive for some bee.

(2) $\forall x \forall y[[F(x) \land B(y)] \rightarrow [A(x, y) \land E(x, y)]]$

Every flower is attractive and easy to find for every bee.

(3) $\forall x[F(x) \rightarrow [\exists y[B(y) \land E(x, y)] \land \exists y[B(y) \land A(x, y)]]]$

Every flower is easy to find for some bee and attractive for some bee.

3(a). Give a recursive definition of the set of palindromes of odd length consisting of symbols "a" and "b". A palindrome is a string that reads the same backwards as forwards and the length of a string is the number of symbols in the string. For example, "a", "aba". "bbbbb" and "baaab" are palindromes of length 1, 3, 5 and 5, respectively. [15]

Let P be the set of palindromes of odd length. Then P is the set that satisfies the following clauses:
Basis Clause: $a \in P$ and $b \in P$.
Inductive Clause: $\forall x[x \in P \rightarrow axa \in P \wedge bxb \in P]$
Extremal Clause: Nothing is in P unless it is obtained by the basis and inductive clauses.

3(b). Using your definition in 3(a), find all palindromes of length 1 and 3. [5]

a, b, aaa, aba, bab, bbb.

4. Which of the following statements are true and which are false ?
A, B and C below represent sets. [30]

(1) $\{1,2\} \times \emptyset = \{< 1, \emptyset >, < 2, \emptyset >\}$: False
(2) $\{\emptyset\} \subseteq \{2\}$: False
(3) $\emptyset \in \{\{1\}, \{2\}, \{\emptyset\}\}$: True
(4) $\{1,2\} = \{1,2,1\}$: True
(5) $(A - B) \cup (A - C) = A - (B \cap C)$: True
(6) If $A \cup B = B$, then $A \cap B = A$: True
(7) $\{1\} \subseteq 2^{\{1,2\}}$: False
(8) $\emptyset \in A$ for every set A: False
(9) If $A \cup B = U$ and $A \cap B = \emptyset$, where U is the universal set, then $B = \overline{A}$: True
(10) $(A - B) \cup (B - C) = A - C$: False
(11) If $(A - B) \cup (B - A) = A \cup B$, then $A \cap B = \emptyset$: True
(12) The cardinality (size) of $\{1,2,1\}$ is 3: False
(13) $\mid A \times B \mid = 6$, if $\mid A \mid = 3$ and $\mid B \mid = 2$: True
(14) The maximum possible value of $\mid A \cap B \mid$ is equal to the smaller of $\mid A \mid$ and $\mid B \mid$: True
(15) $A \times B = B \times A$: False

5. Prove by mathematical induction the following: [20]

$\Sigma_{i=1}^{n} i^3 = (n(n+1)/2)^2$

Basis Step: n = 1. $\Sigma_{i=1}^{1} i^3 = 1^3 = 1$. $(1 * (1+1)/2)^2 = 1$.
Hence LHS = RHS.

Inductive Step:
Induction Hypothesis: $\Sigma_{i=1}^{n} i^3 = (n(n+1)/2)^2$

Then $\Sigma_{i=1}^{n+1} i^3 = \Sigma_{i=1}^{n} i^3 + (n+1)^3$
$= (n(n+1)/2)^2 + (n+1)^3$ by the Induction Hypothesis
$= (n+1)^2(n^2 + 4n + 4)/4$
$= ((n+1)(n+2)/2)^2$