CS 381 Final Exam

May, 2005

1. For the following argument answer the questions below:

$\forall x[\neg P(x) \vee Q(x)]$
$\forall x R(x)$
$\exists x[\neg R(x) \vee P(x)]$

$\exists x Q(x)$

(a) Is the argument logically correct ? [5]
(b) Justify your answer to (a). [15]

2. Negate each of the propositions given below in English. Give a form **other than** simply putting 'not' or 'It is not the case that' in front or anything similar. [15]

(a) He likes it but she doesn't like it.

(b) I can do it only if it doesn't take much time.

(c) Either everyone likes it or everyone doesn't like it.

(d) There is a city that everyone loves.

(e) Everyone likes some book.

3. Prove that if $(A - B) \cup (B - A) = A \cup B$, then $A \cap B = \emptyset$. [15]

4. For an arbitrary binary relation R, prove that $(R^m)^n = R^{mn}$ by induction on n, where m and n are natural numbers. You may use $R^m R^n = R^{m+n}$ without proof, if necessary. [15]

5. Let R be the equality relation on rationals, that is $< a/b, c/d > \in R$ if and only if $ab = cd$, where a, b, c and d are integers.

(a) Give a general form for the elements of the equivalence class that contains 1/2. [5]

(b) Prove that R is an equivalence relation. [15]

6. Which of the following statements are true and which are false ? [15]

(a) If a relation is symmetric and transitive, it is reflexive.
(b) A function is one-to-one only if it has the inverse.
(c) Every totally ordered set has a maximum element.
(d) Every element is related only to itself in a relation only if the relation is an equivalence relation.
(e) For a relation R_1 to be a subset of a relation R_2, it is necessary that R_2 is the transitive closure of R_1.
(f) A Hasse diagram completely describes a partial order.
(g) A topological order is a total order.
(h) $\{\{\emptyset\}\} \subseteq \{\{\emptyset\}, \{\{\emptyset\}\}, \emptyset\}$.
(i) If a function from a set A to a set B is onto, then $\mid A \mid \leq \mid B \mid$.
(j) If a function f is defined as $f(1) = 1$ and $f(2) = 3$, then f has the inverse.