CS 381 Final Exam

December 2005

1. Negate each of the propositions given below in English. Give a form **other than** simply putting $\neg$ or "It is not the case that" in front. [15]

(a) Every picture is faded only if it is old.

(b) Someone likes every flower.

(c) Some flowers are pink, thorny or fragrant.

2. Express the assertions given below as propositions of predicate logic using the following predicates. The universe is **the set of objects.** [15]

$B(x)$: x is bright colored.
$F(x)$: x is a flower.
$R(x)$: x is red.

(a) Not every flower is red.

(b) Every flower is red only if it is bright colored.

(c) Every flower that is bright colored is not red.

3 (a) Express the argument given below as propositions of **predicate logic** using the following symbols: [6]

$E(x)$: x is a computer engineer.
$G(x)$: x is a member of the group.
$M(x)$: x has a Mathematics degree.
$S(x)$: x is a computer scientist.
$W(x)$: x can work on this project.

(b) Check whether or not the reasoning is correct. Give your reasons. [9]

Argument:
Everyone is not a member of the group, is a computer engineer or is a cmputer scientist. Everyone is a computer scientist and has a Mathematics degree only if he/she can work on this project. Sam is not a computer engineer but he has a Mathematics degree. Therefore, if Sam is a member of the group, he can work on this project.

4. Which of the following statements are true and which are false ? [20]

(a) If $A \cap B = B$, then $A \cup B = A$.
(b) If A - B = $\emptyset$, then A = B.
(c) $\{1\} \subseteq \{\{1\}, 2\}$
(d) $1 \in \{\{1\}, 2\}$
(e) $\emptyset \in \{1, 2\}$
(f) If a function from a set A to a set B is onto, then $\mid A \mid \leq \mid B \mid$
(g) $\emptyset \times \{1, 2\} = \emptyset$
(h) $f(x) = 1/(x^2 - 1)$ is a function from the set of natural numbers to the set of real numbers.
(i) A partially ordered set always has the maximum element.
(j) $f(x) = x + 3$ is a onto function from N to N.

5. Prove by mathematical induction the following: [8 each]

(a) $\Sigma_{i=1}^{n} i(2i + 1) = n(n + 1)(4n + 5)/6$

5 (b) The product of any three consecutive natural numbers such as 2*3*4, 10*11*12, etc. is divisible by 6.

6. Prove that if binary relations R and R^2 on a set A are symmetric, R^3 is symmetric. [9]

7. Given the partition {{1,2}, {3}, {4,5,6}} of the set {1,2,3,4,5,6}, answer the following questions: [5 each]

(a) List the ordered pairs of the equivalence relation that produces the partition.

(b) Find the equivalence class [4].