CS 381 Final Exam

April 29, 2004

1. Express the assertions given below as propositions of predicate logic using the following predicates. The universe is **the set of objects.** [9]

$H(x)$: x is happy.
$P(x)$: x is a person.
$W(x)$: x can work.
$F(x, y)$: x is a friend of y.

a) At least one person is happy.

b) Everyone has at least one friend.

c) Everyone can work only if one is happy.

2. Negate the following statements in English. Give a form other than simply putting "not" or "it is not the case that ..." in front: [9]

a) Everyone has at least one friend.

b) Everyone can work only if one is happy.

c) Someone has visited every store in some town.

3. Recursively define the set of wffs of propositional logic.
Use $P_1, P_2, ..., P_n$ for propositional variables. [10]

4. Prove $A \cup (A \cap B) = A$ by proving the set inclusions. [10]

5. Prove by mathematical induction the following:

(a) $\Sigma_{i=1}^{n} i^3 = n^2(n+1)^2/4$. [10]

(b) The product of any three consecutive natural numbers such as 3*4*5, 9*10*11 etc. is divisible by 3. [10]

6. Let R be a relation on the set of ordered pairs of natural numbers defined as $< a, b > R < c, d >$ if and only if $ad = bc$.

(a) Prove that R is an equivalence relation. [10]

(b) Give an example of equivalence class and show two elements in your equivalence class. [5]

7. Which of the following statements are true and which are false ? [12]

(a) A partially ordered set always has the maximum element.
(b) The $\subseteq$ relation on a set of sets is a total order.
(c) If a poset has two maximal elements, then exactly two total orders can be found on it.
(d) The parent-child relation on a set of people is a partial order.
(e) A Hasse diagram represents a poset.
(f) $\leq$ on the set of integers is a well order.

8. Order the following five functions by the "little-oh" relation. Justify your order by limit test. [15]

$4n^{3/2} + 3n + 5$, $n^{1/3}$, $\ln n$, $nlnn + n^{1/2}$, 2^n