CS 381 Solutions to Test II

November 4, 2004

1. Express the assertions given below as a wff of a predicate logic using the following predicates. The universe is the set of objects.[15 points]

> $P(x)$: x is a person.
> $Hp(x)$: x is happy.
> $Ht(x)$: x is healthy.
> $L(x, y)$: x loves y.

(a) Every happy person is healthy.

$\forall x[[H_p(x) \wedge P(x)] \to H_t(x)]$

(b) Some healthy person is not happy.

$\exists x[P(x) \wedge H_t(x) \wedge \neg H_p(x)]$

(c) Everyone is happy only if someone loves him/her.

$\forall x[[H_p(x) \wedge P(x)] \to \exists y[P(y) \wedge L(y, x)]]$

2 (a) Translate the statements given below into wffs of predicate logic.
Use C(x), F(x), P(x) and R(x) to denote "x is colorful", "x is a flower", "x is a plant" and "x is red", respectively, and assume that the universe is the set of all objects. [9]
(b) Derive the conclusion that "Somethings are colorful if they are plants" from the statements below. Show your reasoning. [11]

> "Some things are flowers if they are plants."
> "Some things are red if they are flowers."
> "All flowers are colorful."

(a)
$\exists x[P(x) \to F(x)]$
$\exists x[F(x) \to R(x)]$
$\forall x[F(x) \to C(x)]$

(b)
$\exists x[P(x) \to F(x)]$

$P(a) \to F(a)$ — EI

$\forall x[F(x) \to C(x)]$

$F(a) \to C(a)$ — UI

$P(a) \to F(a)$
$F(a) \to C(a)$

$P(a) \to C(a)$ — Hypothetical Syllogism

$\exists x[P(x) \to C(x)]$ — EG

3. Find the following Cartesian products: [8]

(a) $\emptyset \times \{\{\emptyset\}, \{1\}\}$

$\emptyset$

(b) $\{\{\emptyset\}, \emptyset\} \times \{\emptyset\}$

$\{< \{\emptyset\}, \emptyset >, < \emptyset, \emptyset >\}$

4. Find the power set of the following sets: [8]

(a) $\{\emptyset\}$

$\{\emptyset, \{\emptyset\}\}$

(b) $\{\{\emptyset\}, \emptyset\}$

$\{\emptyset, \{\{\emptyset\}\}, \{\emptyset\}, \{\{\emptyset\}, \emptyset\}\}$

5. Indicate which of the following are true and which are false. [24]

(a) $\{\{1\}\} \in \{1, \{\{1\}\}\}$ True
(b) $\{\emptyset\} \subseteq \{1, \{\emptyset\}\}$ False
(c) $\emptyset \in \{\{x\}, \{\emptyset\}, x\}$ False
(d) $\emptyset \subseteq \{\emptyset, \{x\}\}$ True
(e) If $A \subseteq B$, then $A \cap B = A$. True
(f) $A - (B \cup B) = (A - B) \cup (A - C)$ False
(g) If $A \cap B = \emptyset$, then $A = \emptyset$. False
(h) $A \subseteq (A - B) \cup B$ True

6. A set L of strings composed of symbols a and b is defined recursively as follows:

Basis Clause: $a \in L$ and $b \in L$.
Inductive Clause: For any string x, if $x \in L$, then $xab \in L$.
Extremal Clause: Nothing is in L unless it is obtained by the Basis and Inductive Clauses.

Answer the following questions on L:

(a) Which of the strings ababb, aababb and bababab are in L ? [6]

bababab is in L. The others are not in L.

(b) Can a string in L end in a ? [4]

Yes. a is in L.

(c) Describe in English the kind of strings in L.
What are their characteristics ? [7]

a or b followed by zero or more ab's

(d) Recursively define the language consisting of strings of repeated ab such as ab, abab, ababab, ... [8]

Let us denote this language by S. Then S is the set that satisfies the following three clauses.

Basis Clause: $ab \in S$
Inductive Clause: For any string x, if $x \in S$, then $xab \in S$.
Extremal Clause: Nothing is in S unless it is obtained by the above two clauses.