CS 281 Test II

March 26, 2003

1. Express the assertions given below as a wff of a predicate logic using the following predicates. The universe is the set of objects.[15]

$CH(x)$: x is a check.
$CS(x)$: x is cashable.
$V(x)$: x is valid.

(a) Some checks are not cashable.

(b) All checks are invalid only if they are not cashable.

(c) For a check to be cashable it is necessary that it is valid.

2. Translate the following wffs into English using the given predicates. The universe is the set of objects.[15]

$B(x)$: x is a bee.
$F(x)$: x is a flower.
$L(x, y)$: x loves y.

(a) $\exists x \exists y [B(x) \land F(y) \land L(x, y)]$

(b) $\exists x \forall y [B(x) \land [F(y) \rightarrow L(x, y)]]$

(c) $\neg \forall x \forall y [[B(x) \land F(y)] \rightarrow L(x, y)]$

3. Find the following Cartesian products: [6]

(a) $\{\emptyset\} \times \{1, 3, 5\}$
(b) $\{1, 4\} \times \emptyset$

4. Recursively define each of the following sets:

(a) The set of positive integers T that produces the remainder 2 when divided by 5. [10]

(b) $P = \{3^n - 2 \mid n \text{ is a natural number and } n \geq 1.\}$ [10]

5. Prove $A - (A - B) = A \cap B$ for sets A and B. [14]

6. Indicate which of the following are true and which are false. [16]

(a) $\emptyset \in \{\{x\}, \emptyset, x\}$
(b) $\{x\} \subseteq \{x, \{x\}\}$
(c) $\{x\} \in \{x, \{\{x\}\}\}$
(d) $\emptyset \subseteq \{x, \{x\}\}$

7. Which rules of inference are used to establish the conclusion of the following argument ?

Show your reasoning symbolically. Use C(x), F(x) and P(x) to denote "x is colorful", "x is a flower" and "x is a plant" respectively, and assume that the universe is the set of all objects. [14]

Premises:

"Some plants are flowers."
"All flowers are colorful."

Conclusion:
"Some plants are colorful."

You may use the following table.

Logical Equivalences and Implications

1. $P \Leftrightarrow (P \vee P)$	2. $P \Leftrightarrow (P \wedge P)$
3. $(P \vee Q) \Leftrightarrow (Q \vee P)$	4. $(P \wedge Q) \Leftrightarrow (Q \wedge P)$
5. $[(P \vee Q) \vee R] \Leftrightarrow [P \vee (Q \vee R)]$	6. $[(P \wedge Q) \wedge R] \Leftrightarrow [P \wedge (Q \wedge R)]$
7. $\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$	8. $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$
9. $[P \wedge (Q \vee R)] \Leftrightarrow [(P \wedge Q \vee (P \wedge R)]$	10. $[P \vee (Q \wedge R)] \Leftrightarrow [(P \vee Q) \wedge (P \vee R)]$
11. $(P \vee T) \Leftrightarrow T$	12. $(P \wedge T) \Leftrightarrow P$
13. $(P \vee F) \Leftrightarrow P$	14. $(P \wedge F) \Leftrightarrow F$
15. $(P \vee \neg P) \Leftrightarrow T$	16. $(P \wedge \neg P) \Leftrightarrow F$
17. $P \Leftrightarrow \neg(\neg P)$	18. $(P \rightarrow Q) \Leftrightarrow (\neg P \vee Q)$
19. $(P \leftrightarrow Q) \Leftrightarrow [(P \rightarrow Q) \wedge (Q \rightarrow P)]$	20. $[(P \wedge Q) \rightarrow R] \Leftrightarrow [P \rightarrow (Q \rightarrow R)]$
21. $[(P \rightarrow Q) \wedge (P \rightarrow \neg Q)] \Leftrightarrow \neg P$	22. $(P \rightarrow Q) \Leftrightarrow (\neg Q \rightarrow \neg P)$
1. $P \Rightarrow (P \vee Q)$	2. $(P \wedge Q) \Rightarrow P$
3. $[P \wedge (P \rightarrow Q)] \Rightarrow Q$	4. $[(P \rightarrow Q) \wedge \neg Q] \Rightarrow \neg P$
5. $[\neg P \wedge (P \vee Q)] \Rightarrow Q$	6. $[(P \rightarrow Q) \wedge (Q \rightarrow R)] \Rightarrow (P \rightarrow R)$
7. $(P \rightarrow Q) \Rightarrow [(Q \rightarrow R) \rightarrow (P \rightarrow R)]$	8. $[(P \rightarrow Q) \wedge (R \rightarrow S)] \Rightarrow [(P \wedge R) \rightarrow (Q \wedge S)]$
9. $[(P \leftrightarrow Q) \wedge (Q \leftrightarrow R)] \Rightarrow (P \leftrightarrow R)$	10. $[(P \rightarrow Q) \wedge (R \rightarrow S)] \Rightarrow [(P \vee R) \rightarrow (Q \vee S)])$

Additional Inference Rule: **Conjunction**

$$\begin{array}{c} P \\ Q \\ \hline P \wedge Q \end{array}$$