CS 281 Test II

March 26, 2003

1. Express the assertions given below as a wff of a predicate logic using the following predicates. The universe is the set of objects.[15]

$CH(x)$: x is a check.
$CS(x)$: x is cashable.
$V(x)$: x is valid.

(a) Some checks are not cashable.

$\exists x[CH(x) \wedge \neg CS(x)]$

(b) All checks are invalid only if they are not cashable.

$\forall x[[CH(x) \wedge \neg V(x)] \rightarrow \neg CS(x)]$

(c) For a check to be cashable it is necessary that it is valid.

$\forall x[[CH(x) \wedge CS(x)] \rightarrow V(x)]$

2. Translate the following wffs into English using the given predicates. The universe is the set of objects.[15]

$B(x)$: x is a bee.
$F(x)$: x is a flower.
$L(x, y)$: x loves y.

(a) $\exists x \exists y[B(x) \wedge F(y) \wedge L(x, y)]$

Some bees love flowers.

(b) $\exists x \forall y[B(x) \wedge [F(y) \rightarrow L(x, y)]]$

Some bees love all flowers.

(c) $\neg \forall x \forall y[[B(x) \wedge F(y)] \rightarrow L(x, y)]$

Not every bee loves every flower.

3. Find the following Cartesian products: [6]

(a) $\{\emptyset\} \times \{1, 3, 5\} = \{<\emptyset, 1>, <\emptyset, 3>, <\emptyset, 5>\}$
(b) $\{1, 4\} \times \emptyset = \emptyset$

4. Recursively define each of the following sets:

(a) The set of positive integers T that produces the remainder 2 when divided by 5. [10]

Basis Clause: $2 \in T$
Inductive Clause: If $x \in T$, then $(x + 5) \in T$.
Extremal Clause: Usual clause.

(b) $P = \{3^n - 2 \mid n \text{ is a natural number and } n \geq 1.\}$ [10]

Basis Clause: $1 \in P$
Inductive Clause: If $x \in P$, then $(3x + 4) \in P$.
Extremal Clause: Usual clause.

For Inductive Clause, try to find a way to express $3^{n+1} - 2$ in terms of $3^n - 2$. Note that $3^{n+1} - 2$ is next to $3^n - 2$ as n advances.
Since $3^{n+1} = 3 \times 3^n$, $3^{n+1} - 2 = 3 \times (3^n - 2) + 4$. Hence the element next to x in P is $3x + 4$.

5. Prove $A - (A - B) = A \cap B$ for sets A and B. [14]

$A - (A - B) = A \cap \overline{A \cap \overline{B}} = A \cap (\overline{A} \cup B) = (A \cap \overline{A}) \cup (A \cap B) = \emptyset \cup (A \cap B)$
$= A \cap B$

Alternatively, let x be an arbitrary element.
Then $x \in A - (A - B) \Leftrightarrow x \in A \wedge x \notin (A - B) \Leftrightarrow x \in A \wedge \neg(x \in A \wedge x \notin B)$
$\Leftrightarrow x \in A \wedge (x \notin A \vee x \in B)$
$\Leftrightarrow (x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B)$
$\Leftrightarrow x \in \emptyset \vee x \in (A \cap B)$
$\Leftrightarrow x \in (A \cap B)$.
Hence $A - (A - B) = A \cap B$.

6. Indicate which of the following are true and which are false. [16]

(a) $\emptyset \in \{\{x\}, \emptyset, x\}$: True
(b) $\{x\} \subseteq \{x, \{x\}\}$: True
(c) $\{x\} \in \{x, \{\{x\}\}\}$: False
(d) $\emptyset \subseteq \{x, \{x\}\}$: True

7. Which rules of inference are used to establish the conclusion of the following argument ?

Show your reasoning symbolically. Use C(x), F(x) and P(x) to denote "x is colorful", "x is a flower" and "x is a plant" respectively, and assume that the universe is the set of all objects. [14]

Premises:
"Some plants are flowers."
"All flowers are colorful."

Conclusion:
"Some plants are colorful."

The argument in symbolic form is as follows:

$\exists x[P(x) \wedge F(x)]$
$\forall x[F(x) \rightarrow C(x)]$
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$\exists x[P(x) \wedge C(x)]$

From $\exists x[P(x) \wedge F(x)]$ by Existential Instantiation, there is an object c for which $P(c) \wedge F(c)$ holds.
Hence by Simplification $F(c)$ holds.
By Universal Instantiation from $\forall x[F(x) \rightarrow C(x)]$ $F(c) \rightarrow C(c)$ holds.
This with $F(c)$ by Modus Ponens produces $C(c)$.
Also from $P(c) \wedge F(c)$ by Simplification $P(c)$ holds.
Hence $P(c) \wedge C(c)$ holds.
Hence by Existential Generalization $\exists x[P(x) \wedge C(x)]$ is obtained.