CS 381 Final Exam

May 7, 2003

1. Express the assertions given below as propositions of predicate logic using the following predicates. The universe is **the set of objects.**

$B(x)$: x is bright colored.
$F(x)$: x is a flower.
$R(x)$: x is fragrant.

a) Not every flower is fragrant.

b) Every flower is fragrant only if it is bright colored.

c) Every flower that is bright colored is not fragrant.

2. Negate each of the propositions given below in English. Give a form **other than** simply putting $\neg$ or "It is not the case that" in front.

(a) Grass grows only if there is enough sun.

(b) Someone has visited every state.

(c) Some flowers are pink, thorny and fragrant.

3 (a) Express the argument given below as propositions of **predicate logic** using the following symbols:

$E(x)$: x is a computer engineer.
$G(x)$: x is a member of the group.
$M(x)$: x has a Mathematics degree.
$S(x)$: x is a computer scientist.
$W(x)$: x can work on this project.

(b) Check whether or not the reasoning is correct. Give your reasons.

Argument:
Everyone is not a member of the group, is a computer engineer or is a cmputer scientist. Everyone is a computer scientist and has a Mathematics degree only if he/she can work on this project. Sam is not a computer engineer but he has a Mathematics degree. Therefore, if Sam is a member of the group, he can work on this project.

4. Which of the following statements are true and which are false ?

(a) If each element in the universe either is not in set A or is in set B then A $\subseteq$ B.
(b) If A - B = $\emptyset$, then A = B.
(c) $\{1\} \in \{\{1\}, 2\}$
(d) $1 \subseteq \{\{1\}, 2\}$
(e) $\emptyset \in \{1, 2\}$
(f) $\emptyset \subseteq \{1, 2\}$
(g) $\emptyset \times \{1, 2\} = \emptyset$
(h) $f(x) = 1/(x^2 - 1)$ is a function from N to N, where N is the set of natural numbers.
(i) $f(x) = x^2$ is a one-to-one function from N to N.
(j) $f(x) = 2x + 3$ is a onto function from N to N.

5. Prove by mathematical induction the following:

$\Sigma_{i=1}^{n} i(i+1) = n(n+1)(n+2)/3$

6. Prove that if a binary relation R on a set is symmetric, R^2 is symmetric.

7 (a) Prove that $\Theta(f)$ is an equivalence relation over the set of functions.

(b) Give two elements of the equivalence class $[x^2]$ for the relation $\Theta(f)$.